

Conflict nets: Efficient locally canonical MALL proof nets

Dominic J. D. Hughes
and
Willem Heijltjes

LICS, 6 July 2016

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- 1934 Gentzen: LJ \rightarrow NJ

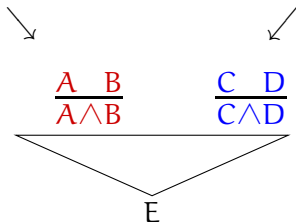
LJ = Intuitionistic sequent calculus

NJ = Intuitionistic natural deduction

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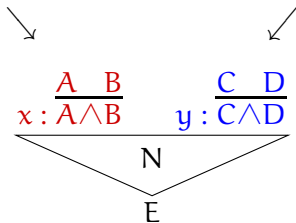
$$\frac{\frac{A, B, C, D \vdash E}{A, B, C \wedge D \vdash E}}{A \wedge B, C \wedge D \vdash E} \approx \frac{\frac{A, B, C, D \vdash E}{A \wedge B, C, D \vdash E}}{A \wedge B, C \wedge D \vdash E}$$



Conflict nets: Efficient locally canonical MALL proof nets

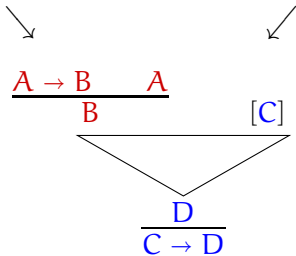
- 1934 Gentzen: $LJ \rightarrow NJ$

$$\frac{\frac{A, B, C, D \vdash E}{A, B, C \wedge D \vdash E}}{A \wedge B, C \wedge D \vdash E} \simeq \frac{\frac{A, B, C, D \vdash E}{A \wedge B, C, D \vdash E}}{A \wedge B, C \wedge D \vdash E}$$



$$N[\langle a, b \rangle / x][\langle c, d \rangle / y]$$

$$\frac{\frac{\frac{}{\vdash A} \quad \frac{B, C \vdash D}{B \vdash C \rightarrow D}}{A \rightarrow B \vdash C \rightarrow D}}{\vdash A \quad B, C \vdash D} \quad \approx \quad \frac{\frac{\frac{}{\vdash A} \quad B, C \vdash D}{A \rightarrow B, C \vdash D}}{A \rightarrow B \vdash C \rightarrow D}}$$



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- 1934 Gentzen: $LJ \dashv\vdash NJ = \wedge$
 - ✓ efficient
 - ✓ canonical for $\rightarrow \wedge$

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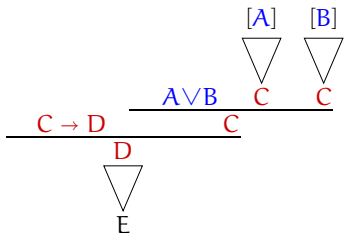
- 1934 Gentzen: $LJ \mapsto NJ = \wedge$
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 - ✗ canonical for $\rightarrow \wedge \vee$

$$\frac{\frac{A \vdash C \quad B \vdash C}{A \vee B \vdash C} \quad D \vdash E}{A \vee B, C \rightarrow D \vdash E}$$

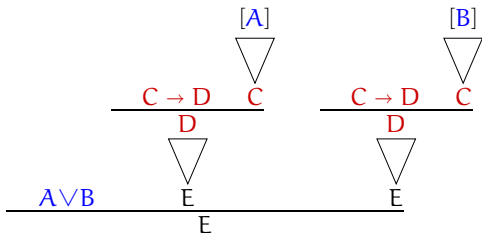
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$$\frac{\frac{A \vdash C \quad D \vdash E}{A, C \rightarrow D \vdash E} \quad \frac{B \vdash C \quad D \vdash E}{B, C \rightarrow D \vdash E}}{A \vee B, C \rightarrow D \vdash E}$$

↓



↓



$$\frac{\frac{A \vdash C \quad B \vdash C}{A \vee B \vdash C} \quad D \vdash E}{A \vee B, C \rightarrow D \vdash E} \sim \frac{\frac{A \vdash C \quad D \vdash E}{A, C \rightarrow D \vdash E} \quad \frac{B \vdash C \quad D \vdash E}{B, C \rightarrow D \vdash E}}{A \vee B, C \rightarrow D \vdash E}$$

↓

$$\frac{\frac{f : C \rightarrow D \quad C}{D} \quad \frac{x : A \vee B \quad \frac{[A] \quad C}{L} \quad \frac{[B] \quad C}{R}}{C}}{E}$$

↓

$$\frac{x : A \vee B \quad \frac{f : C \rightarrow D \quad \frac{D}{N}}{E} \quad \frac{f : C \rightarrow D \quad \frac{[A] \quad C}{L} \quad \frac{[B] \quad C}{R}}{C}}{E}$$

$N[fM / d]$ where

$M = \text{case } v \text{ of } \text{inl } a \mapsto L$
 $\text{inr } b \mapsto R$

$\text{case } v \text{ of } \text{inl } a \mapsto N[fL / d]$
 $\text{inr } b \mapsto N[fR / d]$

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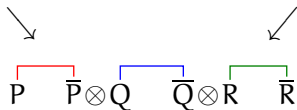
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 - ✓ efficient ✓ canonical for MLL ($\otimes \wp$)

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$$\otimes \frac{\frac{\overline{P, \overline{P}} \quad \overline{Q, \overline{Q}}}{P, \overline{P} \otimes Q, \overline{Q}} \quad \overline{R, \overline{R}}}{P, \overline{P} \otimes Q, \overline{Q} \otimes R, \overline{R}} \otimes \quad \simeq \quad \otimes \frac{\overline{P, \overline{P}} \quad \frac{\overline{Q, \overline{Q}} \quad \overline{R, \overline{R}}}{Q, \overline{Q} \otimes R, \overline{R}}}{P, \overline{P} \otimes Q, \overline{Q} \otimes R, \overline{R}} \otimes$$



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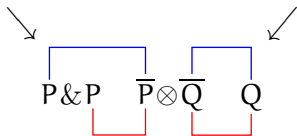
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- 2003 Hughes + Van Glabbeek: $MALL \mapsto$ Slice nets
 - ✗ efficient ✓ canonical

$$\frac{\frac{\overline{P, \overline{P}}}{P \& P, \overline{P}} \quad \overline{\overline{Q}, Q}}{P \& P, \overline{P} \otimes \overline{Q}, Q} \approx \frac{\frac{\overline{P, \overline{P}} \quad \overline{\overline{Q}, Q}}{P, \overline{P} \otimes \overline{Q}, Q} \quad \frac{\overline{P, \overline{P}} \quad \overline{\overline{Q}, Q}}{P, \overline{P} \otimes \overline{Q}, Q}}{P \& P, \overline{P} \otimes \overline{Q}, Q}$$



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- This paper: $MALL \mapsto$ Conflict nets
 - ✓ efficient
 - ✓ *locally* canonical

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 - ✗ efficient ✓ *strongly* canonical
- This paper: $MALL \mapsto$ Conflict nets
 - ✓ efficient ✓ *locally* canonical

All MLL and ALL rule commutations are local

$$\begin{array}{c}
 \otimes \frac{\overline{P, \overline{P}} \quad \overline{Q, \overline{Q}}}{\overline{P, \overline{P} \otimes Q, \overline{Q}}} \\
 \hline
 \otimes \frac{\overline{R, \overline{R}} \quad \overline{S, \overline{S}}}{\overline{R, \overline{R} \otimes S, \overline{S}}} \\
 \hline
 \otimes \frac{\overline{P, \overline{P} \otimes Q, \overline{Q} \otimes (\overline{R} \otimes S), R \mathcal{A} \overline{S}}}{\overline{P \mathcal{A} (\overline{P} \otimes Q), \overline{Q} \otimes (\overline{R} \otimes S), R \mathcal{A} \overline{S}}}
 \end{array}
 \rightarrow
 \begin{array}{c}
 \otimes \frac{\overline{P, \overline{P}} \quad \overline{Q, \overline{Q}}}{\overline{P, \overline{P} \otimes Q, \overline{Q}}} \quad \otimes \frac{\overline{R, \overline{R}} \quad \overline{S, \overline{S}}}{\overline{R, \overline{R} \otimes S, \overline{S}}} \\
 \hline
 \otimes \frac{\overline{P, \overline{P} \otimes Q, \overline{Q} \otimes (\overline{R} \otimes S), R, \overline{S}}}{\overline{P, \overline{P} \otimes Q, \overline{Q} \otimes (\overline{R} \otimes S), R \mathcal{A} \overline{S}}} \\
 \hline
 \otimes \frac{\overline{P, \overline{P} \otimes Q, \overline{Q} \otimes (\overline{R} \otimes S), R \mathcal{A} \overline{S}}}{\overline{P \mathcal{A} (\overline{P} \otimes Q), \overline{Q} \otimes (\overline{R} \otimes S), R \mathcal{A} \overline{S}}}
 \end{array}$$

One MALL rule commutation is not local

$$\frac{\frac{\frac{\Pi}{\vdots} \quad \frac{B, \Delta, C \quad B, \Delta, D}{B, \Delta, C \& D}}{\Gamma, A \otimes B, \Delta, C \& D} \otimes \&}{\Gamma, A \otimes B, \Delta, C \& D} \otimes \rightarrow \otimes \frac{\frac{\frac{\Pi}{\vdots} \quad \frac{\Gamma, A \quad B, \Delta, C}{\Gamma, A \otimes B, \Delta, C}}{\Gamma, A \otimes B, \Delta, C} \otimes \frac{\frac{\Pi}{\vdots} \quad \frac{\Gamma, A \quad B, \Delta, D}{\Gamma, A \otimes B, \Delta, D}}{\Gamma, A \otimes B, \Delta, D} \otimes}{\Gamma, A \otimes B, \Delta, C \& D} \&$$

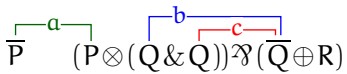
Local/strong canonicity

- *Local canonicity*
Invariance under *local* rule commutations
- *Strong canonicity*
Invariance under *all* rule commutations

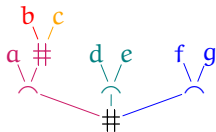
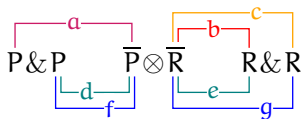
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Conflict net 1



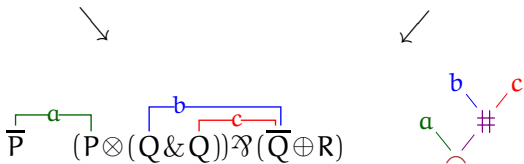
Conflict net 2



Conflict nets are locally canonical

$$\frac{\frac{\frac{\overline{P}, P}{\overline{P}, P} \quad \frac{\overline{Q}, \overline{Q} \quad \overline{Q}, \overline{Q}}{Q \& Q, \overline{Q}} \&}{\overline{P}, P \otimes (Q \& Q), \overline{Q}} \otimes}{\overline{P}, P \otimes (Q \& Q), \overline{Q} \oplus R} \oplus_1}{\overline{P}, (P \otimes (Q \& Q)) \wp (\overline{Q} \oplus R)} \wp$$

$$\simeq \frac{\frac{\frac{\overline{P}, P}{\overline{P}, P} \quad \frac{\overline{Q}, \overline{Q}}{Q, \overline{Q} \oplus R} \oplus_1 \quad \frac{\overline{Q}, \overline{Q}}{Q, \overline{Q} \oplus R} \oplus_1}{Q \& Q, \overline{Q} \oplus R} \&}{\overline{P}, P \otimes (Q \& Q), \overline{Q} \oplus R} \otimes}{\overline{P}, (P \otimes (Q \& Q)) \wp (\overline{Q} \oplus R)} \wp$$



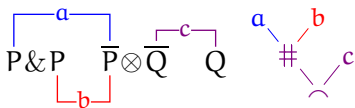
Conflict nets: efficient rather than strongly canonical

$$\frac{\frac{\overline{P, \overline{P}}^a \quad \overline{P, \overline{P}}^b}{P \& P, \overline{P}} \quad \overline{Q, Q}^c}{P \& P, \overline{P} \otimes \overline{Q}, Q}$$

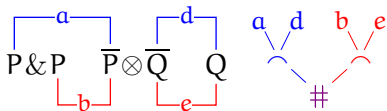
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$$\frac{\frac{\overline{P, \overline{P}}^a \quad \overline{Q, Q}^d}{P, \overline{P} \otimes \overline{Q}, Q} \quad \frac{\overline{P, \overline{P}}^b \quad \overline{Q, Q}^e}{P, \overline{P} \otimes \overline{Q}, Q}}{P \& P, \overline{P} \otimes \overline{Q}, Q}$$

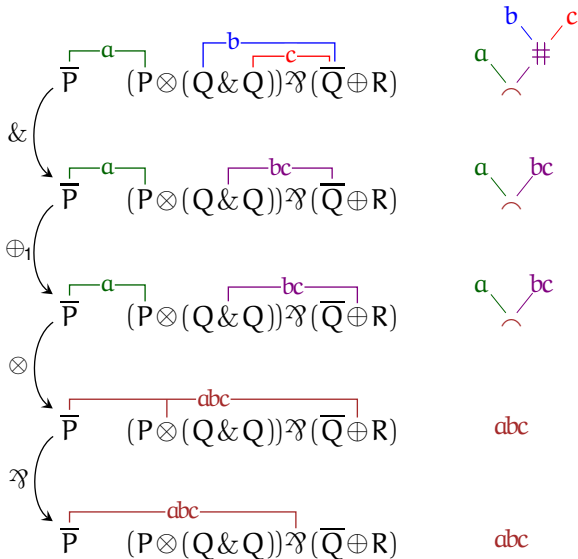
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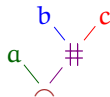
Coalescence correctness (generalizing MLL contractibility)



$$\overline{Q, \overline{Q}}^{bc} \quad \overline{Q, \overline{Q}}^c$$

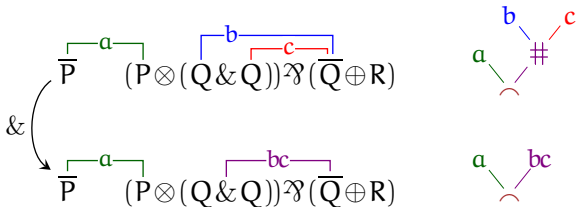
$$\overline{\overline{P}, P}^a$$

$$\overline{P} \quad (P \otimes (Q \& Q)) \wp (\overline{Q} \oplus R)$$

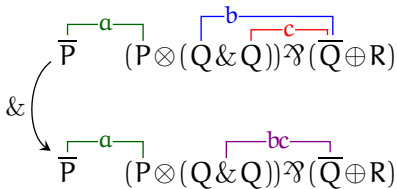


$$\overline{Q, \overline{Q}}^{bc} \quad \overline{Q, \overline{Q}}^c$$

$$\overline{\overline{P}, P}^a$$



$$\overline{\overline{P}, P}^a \quad \frac{\overline{Q, \overline{Q}} \quad \overline{Q, \overline{Q}}}{Q \& Q, \overline{Q}}^{bc}$$



$$\overline{\overline{P}, P}^a \quad \frac{\overline{Q, \overline{Q}} \quad \overline{Q, \overline{Q}}}{Q \& Q, \overline{Q}}^{bc}$$

$$\overline{P} \quad (P \otimes (Q \& Q)) \wp (\overline{Q} \oplus R)$$



$$\overline{\overline{P}, P}^a \quad \frac{\overline{Q, \overline{Q}} \quad \overline{Q, \overline{Q}}}{Q \& Q, \overline{Q}}^{bc}$$

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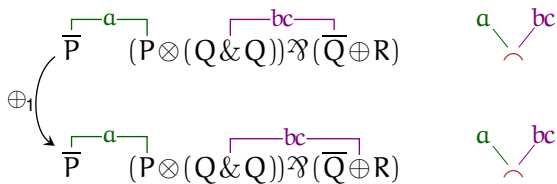


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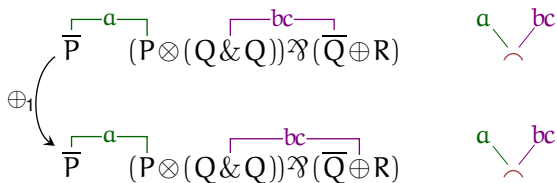
$$\overline{P} \quad (P \otimes (Q \& Q)) \wp (\overline{Q} \oplus R)$$



$$\overline{\overline{P}, P}^a \quad \frac{\overline{Q, \overline{Q}} \quad \overline{Q, \overline{Q}}}{Q \& Q, \overline{Q}}^{bc}$$



$$\overline{\overline{P}, P}^a \quad \frac{\overline{Q, \overline{Q}} \quad \overline{Q, \overline{Q}}}{\overline{Q \& Q, \overline{Q}}}^{bc}$$



$$\overline{\overline{P}, P}^a \quad \frac{\overline{Q, \overline{Q}} \quad \overline{Q, \overline{Q}}}{Q \& Q, \overline{Q}}^{bc} \\ \overline{Q \& Q, \overline{Q} \oplus R}$$

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$$\overline{\overline{P}, P}^a \quad \frac{\overline{Q, \overline{Q}} \quad \overline{Q, \overline{Q}}}{Q \& Q, \overline{Q}}^{bc}$$

$$\overline{Q \& Q, \overline{Q} \oplus R}^{bc}$$

$$\overline{P} \quad (P \otimes (Q \& Q)) \wp (\overline{Q} \oplus R)$$

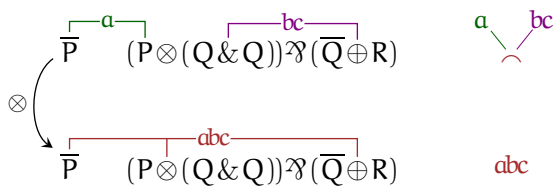


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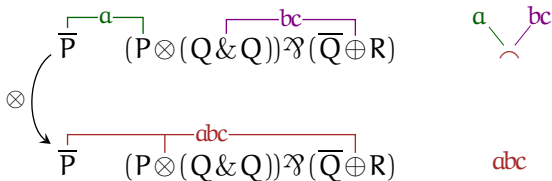
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$$\begin{array}{c}
 \overline{Q, \overline{Q}} \quad \overline{Q, \overline{Q}} \\
 \hline
 Q \& Q, \overline{Q} \\
 \hline
 \overline{P}, P \quad Q \& Q, \overline{Q} \oplus R \\
 \hline
 \overline{P}, P \otimes (Q \& Q), \overline{Q} \oplus R \quad abc
 \end{array}$$



$$\begin{array}{r}
 \overline{Q, \overline{Q}} \quad \overline{Q, \overline{Q}} \\
 \hline
 Q \& Q, \overline{Q} \\
 \overline{P, P} \quad \overline{Q \& Q, \overline{Q} \oplus R} \\
 \hline
 \overline{P, P \otimes (Q \& Q), \overline{Q} \oplus R} \quad abc
 \end{array}$$

$$\overline{P} \quad (P \otimes (Q \& Q)) \mathcal{A} (\overline{Q} \oplus R) \quad abc$$

$$\begin{array}{r}
 \overline{Q, \overline{Q}} \quad \overline{Q, \overline{Q}} \\
 \hline
 Q \& Q, \overline{Q} \\
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 \overline{Q, \overline{Q}} \quad \overline{Q, \overline{Q}} \\
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$$\begin{array}{r}
 \overline{P} \quad (P \otimes (Q \& Q)) \wp (\overline{Q} \oplus R) \quad abc \\
 \wp \curvearrowright \\
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 \end{array}$$

$$\begin{array}{r}
 \overline{Q, \overline{Q}} \quad \overline{Q, \overline{Q}} \\
 \hline
 Q \& Q, \overline{Q} \\
 \overline{P}, P \quad \overline{Q \& Q, \overline{Q} \oplus R} \\
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 \overline{P} \quad \overline{(P \otimes (Q \& Q)) \wp (\overline{Q} \oplus R)} \quad abc \\
 \wp \curvearrowright \\
 \overline{P} \quad \overline{(P \otimes (Q \& Q)) \wp (\overline{Q} \oplus R)} \quad abc
 \end{array}$$

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 \overline{Q, \overline{Q}} \quad \overline{Q, \overline{Q}} \\
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 \hline
 \overline{P}, P \otimes (Q \& Q), \overline{Q} \oplus R \\
 \hline
 \overline{P}, (P \otimes (Q \& Q)) \wp (\overline{Q} \oplus R) \quad abc
 \end{array}$$

$$\overline{P} \quad \overbrace{(P \otimes (Q \& Q)) \wp (\overline{Q} \oplus R)}^{abc} \quad abc$$

