

Presketches: Algebra without algebras via categories without functors

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Abstract

Bypassing the traditional separation of theory and model, we introduce the notion of *presketch* as a pointed category, one with a set of distinguished objects as its points or types. Algebras and homomorphisms arise simply as the objects and morphisms of a presketch. As a generalization of the completion of the rationals to the reals, a presketch is *full* when it densely embeds its points, and *complete* when it is full and maximal up to equivalence. Every complete presketch is a topos by virtue of being equivalent to a presheaf category, and every presheaf category arises as a complete presketch. The category of models of an Ehresmann sketch arises as a full subcategory of a presketch consisting of those algebras respecting specified limits and colimits; as such the models of a sketch in general do not form a topos.

The passage to a disketch as a category with two sets of distinguished objects, positive and negative, or types and properties, generalizes the passage from sets (more generally the objects of the ambient enriching category V) to Chu spaces by interpreting the morphisms from a type to an algebra A as its individuals of that type, and those from A to a property as the local states of observation in A of that property. C.I. Lewis's problematic qualia (1924) are accounted for in this framework simply as those entities that are ambiguously an individual and a state. As often happens, the previous absence of any mathematically plausible account of qualia might explain the strongly partisan division of philosophers into qualiaphiles and qualiaphobes.

Presketches exploit the Yoneda Lemma to move functors and natural transformations out of the passenger compartment and under the bonnet where they can be accessed as needed without intruding unnecessarily on the working mathematician's day-to-day use of algebra.