The Logic of Global Warming
A bitter pill

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What is climate change?

Not all hot air.

Detrimental trends in environmental parameters, most notably:

1. Unsustainable population growth.
2. Accumulation of hazardous materials: lead, mercury, CFCs, ...
3. Global warming: rapid accumulation of heat by the planet. Focus of this talk.
4. Ocean acidification: rapid decrease in ocean pH.
5. Depletion of water, food, and fuel
6. More frequent severe storms and earthquakes
7. Rising sea levels
8. Mass extinctions → less biodiversity → slow or no recovery

What is the root cause?

Product of rate of human population growth with rate of technology growth.

Last 300 years:

Population increase: 1 order of magnitude ($\approx 10\times$)

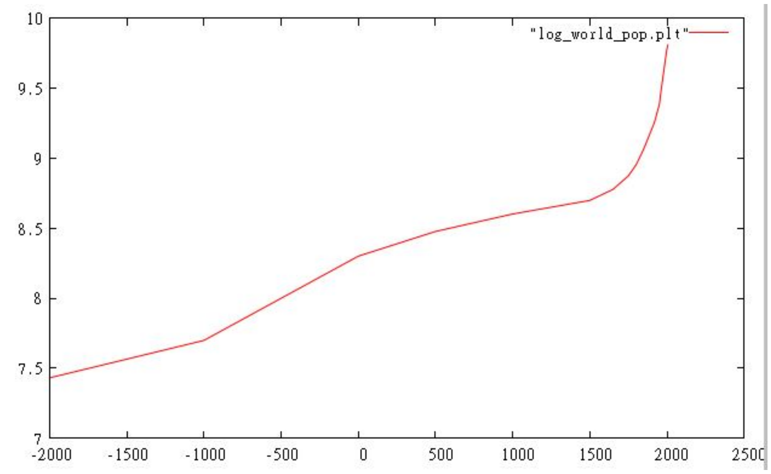
(600 million in 1700, 6.9 billion today)

Technology increase: 1 order of magnitude ($\approx 10\times$) *per capita*

Resulting impact:

Two orders of magnitude ($\approx 100\times$)

Projected growth



Superexponential growth → next $100\times$ increase $<$ 300 years away

Nonhuman causes of climate change

Not all our fault.

100 million years: occasional devastating climate changes: Paleocene-Eocene Thermal Maximum (PETM), Azolla event.

100 thousand years: glaciations, 10° range

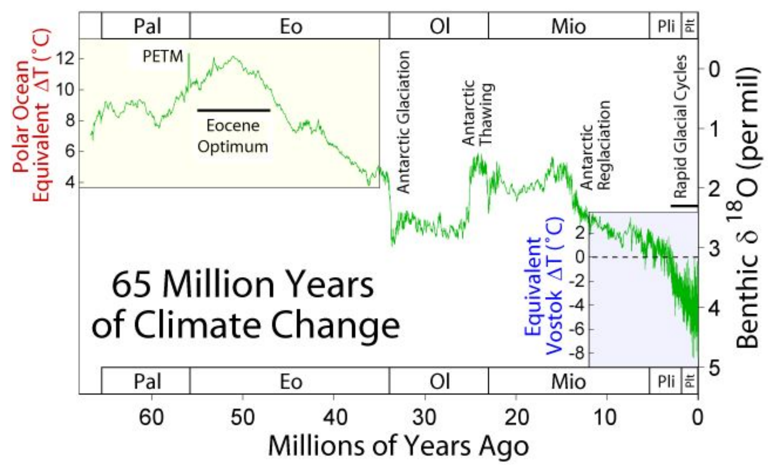
100 years (centennial): ocean oscillations 0.6° range (±0.3° C), of greatest relevance to climate change

10 years (decadal): El Nino, sunspots

1 year (annual): seasons (polar: arctic + temperate)

1 day (diurnal): surface (< 2 km) oscillation (equatorial: temperate + tropical)

Last 65M years



Mechanisms of climate change

1. Heat

reductions in Earth's reflectivity and emissivity.

Reflectivity reduced by increasing atmospheric soot (..., Jacobson)

Emissivity reduced by increasing CO2 and other greenhouse gases (Tyndall, Arrhenius)

(Will argue later that this combination constitutes a nice counterexample to the 2nd law of thermodynamics.)

2. Acidity

Carbonate: increasing CO2 enters ocean and shifts equilibrium of

$$\mathrm{CaCO_3 + CO_2 + H_2O \;\rightleftharpoons\; Ca^{2+}(aq) + 2\ HCO_3^-(aq)}$$

Consumes carbonate faster than coasts can replenish it.

Crustaceans unable to adapt in time.

Logic of global warming

Global warming is a bitter pill.

A great many people today are still reluctant to accept it.

Most are too ill-informed to be able to tell who to believe.

More careful reasoning about the thermal impacts of rapidly increasing population and technology may help at least the technically grounded.

Simplifying the reasoning where possible may extend the understanding to a wider segment of the population.

There will always remain those who are unable to follow the detailed reasoning, or who wilfully reject the reasoning and substitute their own.

Temperature of the celestial sphere

Earth's orbital radius $R_o = 149.6$ gm.

Area of celestial sphere at distance R_o: $4\pi R_o^2$.

Sun's radius $r_s = 0.696$ gm.

Area of Sun (as a "hole" in the sky): πr_s^2.

Sky-Sun ratio $= 4\pi R_o^2/\pi r_s^2 = (2R_o/r_s)^2 = (2 * 149.6/.696)^2 = 184800$

Stefan-Boltzmann law: radiated heat $Q \propto T^4$.

$184800 = 20.73^4$.

Hence the Sun at 5778 K is *heat-equivalent* to the sky at $5778/20.73 =$ 278.7 K, or 5.55° C.

But not *spectrally equivalent*: 20× shorter wavelengths.

(Possible to derive a paradox from these circumstances.)

Impact on Earth

On average, Earth is in thermal equilibrium with the sky.

Hence its temperature as determined *naively* by exchange of heat with the sky is 278.7 K, = 5.5° C.

But 0.3 of that heat is reflected (albedo) as shortwave radiation.

This leaves only $0.7 = 0.915^4$ to be absorbed by earth.

In equilibrium this absorbed heat is radiated back to the sky.

Hence the apparent temperature of Earth, as a black body radiator seen from space, is 278.7*0.915 = 255 K.

Averaging temperatures annually or longer, the coldest point on Earth is the equatorial tropopause, altitude 17 km, at 218 K or −55° C. Polar tropopause is 228 K.

Instantaneously, much colder points are possible, e.g. either pole shortly after its midwinter.

The promised paradox

Consider a highly reflective solid metal ball initially at a uniform 255 K throughout.

Place the ball at the centre of a much larger sphere maintained at 278.7 K.

With no other source of heat than the outer sphere, can the temperature of any portion of the ball fall?

Does this violate the 2nd law of thermodynamics? Explain your reasoning.

Food for thought Does the annually averaged temperature of 218 K at the equatorial tropopause violate the 2nd law of thermodynamics? Explain your reasoning.

Surface temperature

Although the apparent temperature of Earth as seen from space is 278.7*0.915 = 255 K, the average *surface* temperature is *nominally* 288 K (15° C), 33° higher. (*Actual* average depends on how "average temperature" is defined.)

This difference is due primarily to two things.

(i) The insulating qualities of atmospheric greenhouse gases (GHGs). These are gases pass incoming shortwave (solar-wavelength) radiation while absorbing outgoing longwave (terrestrial-wavelength) radiation. The principal greenhouse gases are H_2O (as invisible water *vapour*) and CO_2 (carbon dioxide), with methane (CH_4) in the wings.

(ii) Clouds. Water droplets that scatter, absorb, and radiate differently from GHGs. Average altitude much lower than that of GHGs, effectively warmer by 10-30°, easily observed with a $40 infrared thermometer.

Variations in each of GHG levels and cloud cover, averaged annually or decadally, have a large impact on surface temperature.

Quantity of atmospheric CO_2

Imagine atmospheric CO2 freezing and falling to Earth to form a layer of dry ice.

Each 1‰ (part per thousand) adds 1 cm of dry ice.

Atmospheric CO2 is currently at 0.4‰ (0.039%).

Hence 0.4 cm (4 mm) of dry ice.

This is the thickness of a sheet of glass.

Glass and CO_2 trap longwave radiation (LR) to a similar degree.

So CO_2 can be expected to trap roughly the same amount of heat as a sheet of glass.

For both glass and CO_2, we can calculate how much heat is so trapped.

CO_2 Accumulation

The consensus figure for CO_2 a few centuries ago is 280 ppmv (0.28‰ or 0.028%).

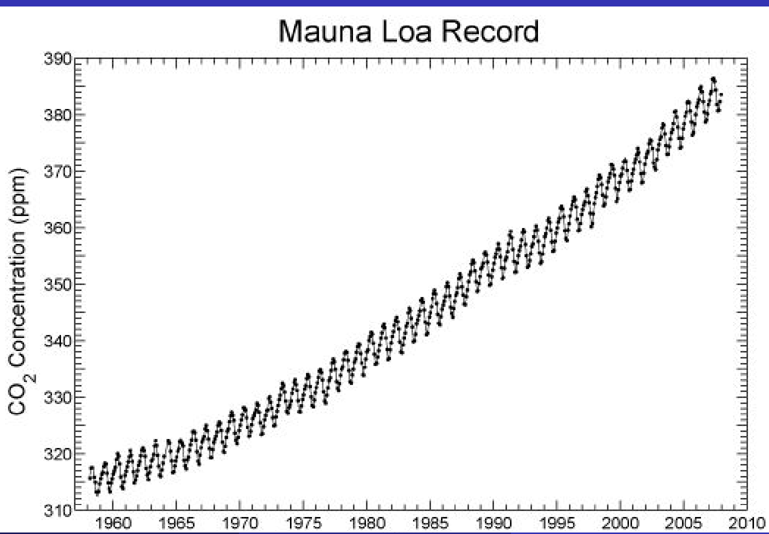
1960: 317 ppmv, increasing at 0.25%/yr.

2011: 390 ppmv = 592 ppmm, increasing at 0.6%/yr.

Atmosphere: 5140 teratonnes. Hence CO2 mass is 5.14*592 = 3044 gigatonnes, increasing at 3044*.006 = 18.3 Gt/yr.

Humans emit 30 Gt/yr, of which nature appears to be removing about half.

Keeling curve



Hofmann's law

David Hofmann, late of NOAA ESRL, has proposed a closed-form formula for CO_2:

$$C(y) = 280 + 2^{\frac{y-1790}{32.5}}\ \mathrm{ppmv}$$

This is consistent with the accepted historical natural base of 280 ppmv.

It is a good fit to the Keeling curve.

It is justifiable in terms of approximately exponential growths in population and per capita fuel consumption, each doubling about every four decades.

Alternatively it can be justified using

Marland and Boden, "Global CO2 Emissions from Fossil-Fuel Burning, Cement Manufacture, and Gas Flaring: 1751-2007", Oak Ridge CO_2 Information Analysis Center, 2010.

Planck's Law of black body radiation

Percentile	.01%	.1%	1%	10%	20%	**25.0%**	30%	40%	**41.8%**	50%	60%	**64.6%**	70%	80%	90%	99%	99.9%	99.99%
Sun λ (nm)	157	192	251	380	463	**502**	540	620	**635**	711	821	**882**	967	1188	1623	3961	8933	19620
288 K planet λ (μm)	3.16	3.85	5.03	7.62	9.29	**10.1**	10.8	12.4	**12.7**	14.3	16.5	**17.7**	19.4	23.8	32.6	79.5	179	394

90% of the radiation from a black body of temperature 288 K is in the frequency range 231 cm^{-1} (43.30 μ) to 1528 cm^{-1} (6.54 μ).

Arrhenius's law

Surface temperature increases logarithmically with level of atmospheric CO_2.

Climate sensitivity is the increase in degrees C per doubling of CO_2.

1896: Arrhenius proposed a sensitivity of 4-5 °/dbling.

1904: Revised downwards to 1.6 °/dbling.

Arrhenius's law is not derivable in the way that say altitude depends logarithmically on pressure. Rather it is a consequence of how the relevant absorption lines of CO2 are distributed according to strength.

$\frac{1}{2}$-closed CO_2 absorption lines per CO_2 doubling

0.8 ppm	3	+3
1.5 ppm	17	+14
3.0 ppm	38	+21
6.1 ppm	54	+16
12.2 ppm	64	+9
24.4 ppm	77	+13
48.8 ppm	114	+37
97.5 ppm	192	+78
195.0 ppm	250	+59
390.0 ppm	311	+61

0.08%	382	+71
0.16%	467	+85
0.31%	527	+59
0.62%	659	+132
1.25%	813	+154
2.50%	1027	+214
4.99%	1219	+192
9.98%	1420	+201
19.97%	1679	+259

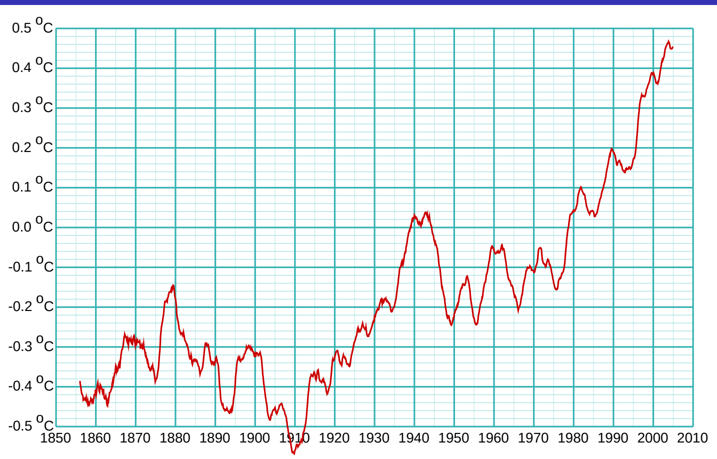
Distribution of lines according to what they pass today

The principal species of CO_2, $^{12}C^{16}O_2$, has 28,039 absorption lines in the middle 90% of the thermal spectrum, from 231 cm^{-1} to 1528 cm^{-1}.

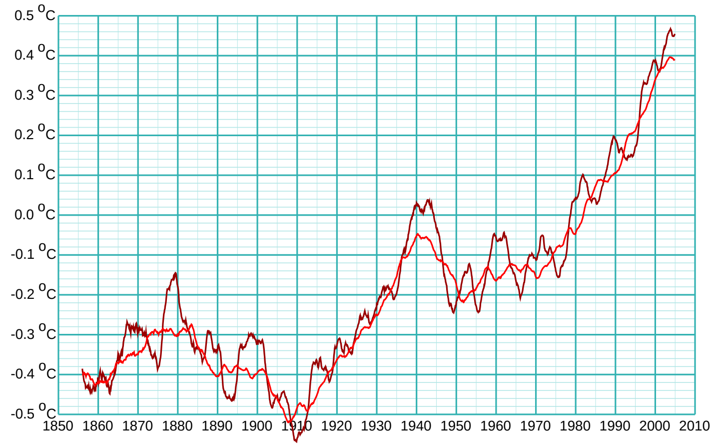
The table below divides up these up into the those lines passing less than 0.1, 0.2, etc. of the radiation at their wavelength, at the present CO_2 level of 390 ppmv.

0.1	207
0.2	32
0.3	23
0.4	23
0.5	26
0.6	31
0.7	34
0.8	63
0.9	70
1.0	27530

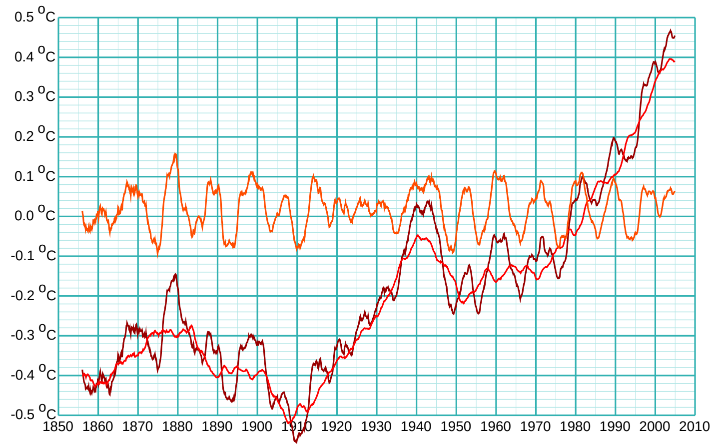
2-yr moving average of global temperature 1850-now



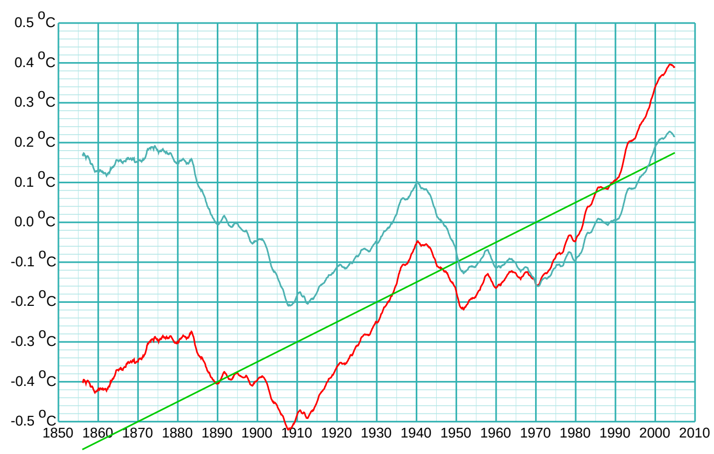
Further smoothing to 12-yr moving average



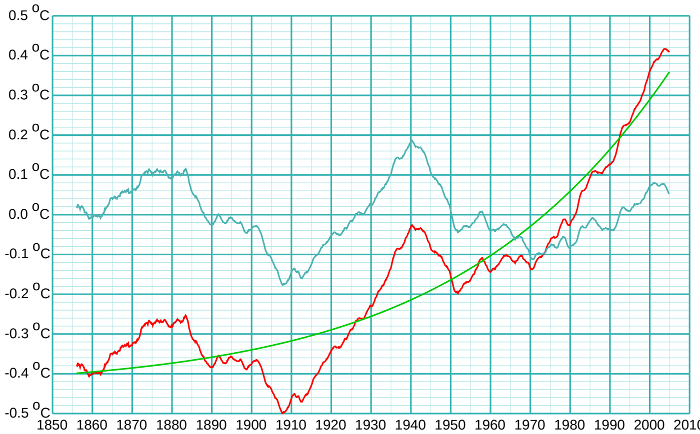
2-yr avg minus 12-yr avg, associated with solar cycles?



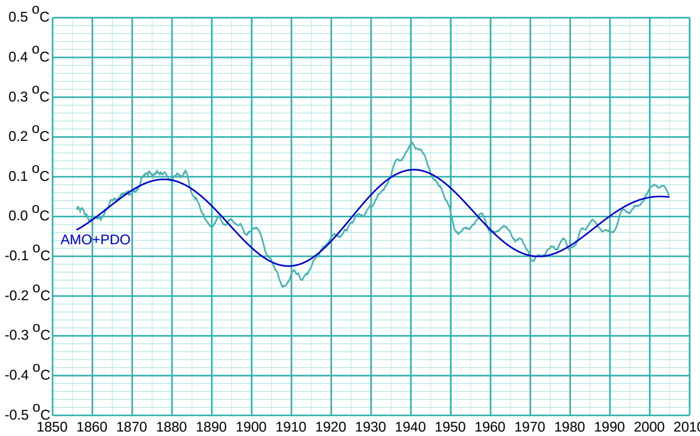
Linear fit (LIN) to 12-yr avg TMP: TMP - LIN in blue



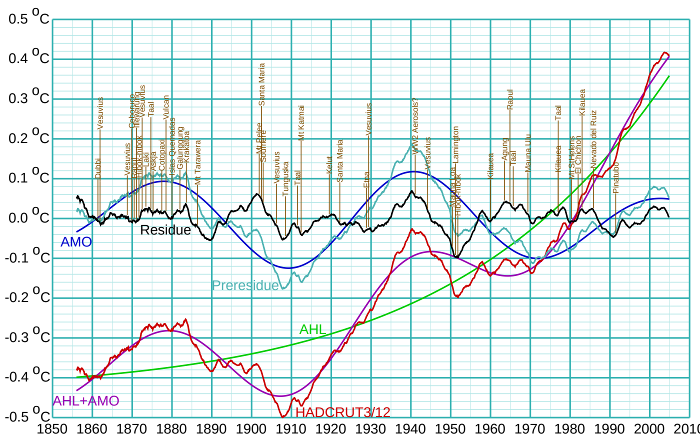
Arrhenius-Hofmann Law (AHL): TMP - AHL in blue



Sinusoidal fit (AMO) to TMP - AHL



AHL + AMO as model of long-term climate change



HADCRUT3/12: global temperature, 1850-2011, unadjusted, smoothed to 12-year moving average

AHL(y): Arrhenius-Hofmann Law, 1.837 lb$(280 + 2^{(y - 1790)/32.5})$

Preresidue = HADCRUT3/12 - AHL. $r^2 = 0.870$

AMO(y): Atlantic Multidecadal Oscillation correlate, 0.0660 $(\sin(2\pi (y - 1925)/56) + \sin(2\pi (y - 1925)/75))$

30 years ago

What would this approach have projected 30 years ago, in 1981?

At that time global temperature had fluctuated with no promise of any increase in temperature.

In fact temperature flattened out in the past couple of decades.

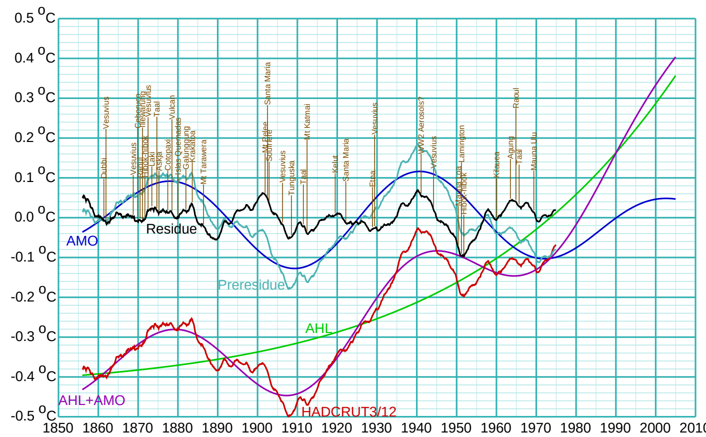
Yet if we had performed a least squares fit of this model to the data up to 1981, it would have projected almost exactly the rise that we witnessed over the past three decades.

There are two contributors.

(i) By 1981 there was enough CO2 data for Hofmann's raised-exponential model to forecast the coming rise in AHL, the contribution of CO2 to global warming.

(ii) By 1981 there was enough temperature data to separate the 56-year and 75-year period ocean oscillations, and to estimate their amplitude with essentially the same result as with the additional 30 years of data.

30 years ago and projection



HADCRUT3/12: global temperature, 1850-1981, unadjusted, smoothed to 12-year moving average

AHL(y): Arrhenius-Hofmann Law, 1.823 lb$(280 + 2^{(y - 1790)/32.5})$

Preresidue = HADCRUT3/12 - AHL. $r^2 = 0.575$

AMO(y): Atlantic Multidecadal Oscillation correlate, 0.0664 $(\sin(2\pi (y - 1925)/56) + \sin(2\pi (y - 1925)/75))$

Greenhouses

Greenhouses stay warm by retaining the warm air with their walls and roof. Earth stays warm by retaining the warm air with gravity.

But how does the air get warm in the first place? Cf. Louvre, 's story.

Wood's 1909 experiment, questioned by Abbot 5 months later.

Easy experiment: Wrap two sheets of glass in plastic wrap, put black cardboard behind one, white behind the other. Leave in sunlight for 5 minutes. Result: glass over black cardboard gets hotter quicker.

Duplicating Wood's experiment.

See http://boole.stanford.edu/WoodExpt