Harmonic Predictive Control

as a variant of PID control

Vaughan Pratt

Comp. Sci. & Hopkins Marine Station
Stanford University

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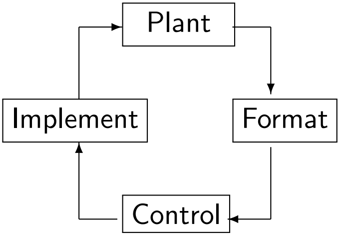
ICCAT, Madrid

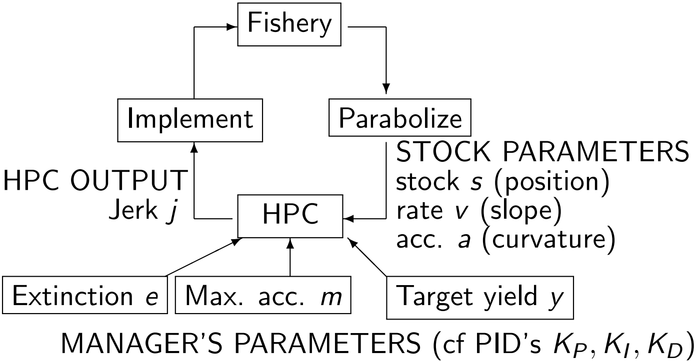
The generic control paradigm



Adaptation to fishery management (clockwise):

Plant	The fishery
Format	Reduce all fishery data to the form expected by Control
Control	The abstract management strategy (PID, MPC, HPC, etc.)
Implement	Convert abstract recommendation to concrete quotas, tasks, etc.

Harmonic Predictive Control



Aspects of HPC, I:

Parabolize	Reduce all fishery data to a parabolic trajectory
Implement	Convert third derivative (jerk j) to concrete quotas, tasks, etc.

The HPC algorithm

HPC "continually" computes two auxiliary variables, Proximity p and Headroom h, and outputs their product ph as Jerk j.

In the following [x, y] denotes a selection conditioned on whether the stock is collapsing ($v < 0$) or recovering ($v > 0$).

$$p = v/([e, y] - s); \qquad \text{(Proximity)}$$

$$h = [m - a, -(3a + 2vp)]; \qquad \text{(Headroom)}$$

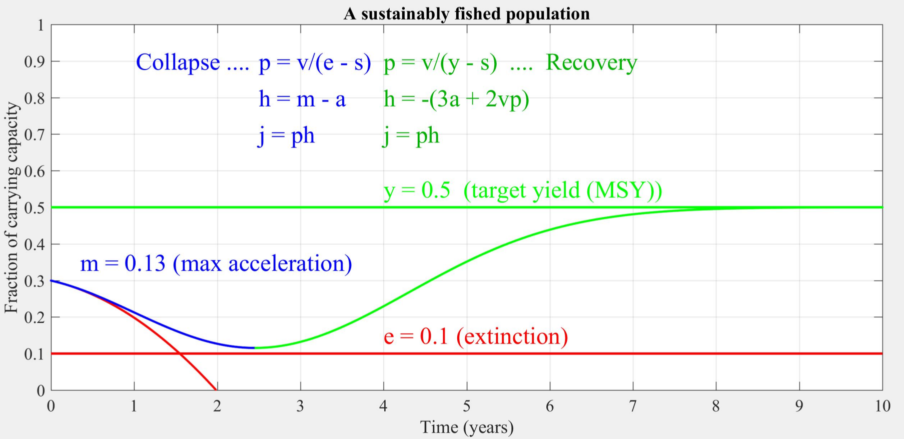
$$j = ph; \qquad \text{(Jerk)}$$

This algorithm "bends" a parabola into a [sinusoid, gaussian].

When collapsing, as the stock s approaches extinction e, proximity p increases. But as acceleration approaches its allowed maximum m, headroom h decreases. If m is sufficiently large stock s will turn around before it reaches extinction e.

Recovery is more mysterious.

Example behavior



Features of HPC

Predictive At every iteration HPC forgets its history and recalculates jerk j from scratch based only on the Stock Parameters s, v, a which vary at every iteration and the Manager's Parameters e, m, y which change more slowly if at all.

Bang-bang It follows a sinusoidal trajectory during collapse and switches abruptly to a gaussian trajectory during recovery.

C2-continuous (Gentle) No curvature discontinuities. (Typical bang-bang controllers are not even C1-continuous.)

As simple as PID Both have three stock parameters and three manager's parameters, and the arithmetic is of comparable complexity.

Nonlinear control Whereas PID is a linear combination of P, I, D, HPC is a nonlinear (but rational) function of its parameters.

Optimal Stock s reaches extinction e if and only if acceleration a reaches its maximum m. But even when s drops below e HPC wishfully predicts s will eventually recover! (But may be wrong about that.)

Handling noise and delay

Formatting should aim to produce stock parameters that vary smoothly.

One way to smooth out noise is with a Kalman filter. Given the coarse sampling typical of fisheries, Kasper Kristensen's TMB as presented yesterday by Anders Nielsen should be a significant improvement. (Is it non-Bayesian as claimed? Let the statisticians argue that one.)

Delays in getting the stock parameters to the controller may lead to controller output that would have been useful if applied earlier but that may need to be quite different now.

One way to deal with this is to run the controller in "prediction mode" up to the present time assuming no further disturbances and use that j as the output. As more current data arrives gradually modify the predicted trajectory to bring it continuously into agreement with what the more current data indicates it should be.

Why "Harmonic"?

In both modes the trajectory has the general harmonic form $s = \Re(e^{\beta(t)}) + s_0$ where $\beta(t)$ is a polynomial and s_0 is an offset.

While collapsing the trajectory is *sinusoidal*:
$\beta(t) = \beta_0 + \beta_1 t$ is a complex linear function of time where

- $\beta_0 = \ln(r) + \imath\varphi$ is complex and gives log amplitude and phase
- $\beta_1 = \imath\omega$ is imaginary and gives the frequency
- s_0 is a real satisfying $s_0 > e$. It contributes the fourth DOF.

Hence four parameters: frequency ω, amplitude r, phase φ, offset s_0.

While recovering, the trajectory is *Gaussian*:
$\beta(t) = \beta_0 - \frac{1}{2}((t - \beta_1)/\beta_2)^2$ is a real quadratic function of time, where

- $\beta_0 = \ln(r)$ is log amplitude as in avoid mode
- $\beta_1 = t_0$ is central time ("mean"), corresponding to φ
- $\beta_2 = 1/\omega$ is angular period ("std dev"), ω as in avoid mode
- $s_0 = y$, a manager's parameter. The three parameters $\beta_0, \beta_1, \beta_2$ constitute the three DOFs of the gaussian.

Derivation of the algorithm's formulas

Sinusoids and gaussians are solutions of differential equations, respectively second order and first order.

Taking the derivative of the sinusoid's equation yields the formula $j = (v/(e-s))(m-a) = ph$.

Taking the derivative of the gaussian's equation twice yields the formula $j = (v/(y-s))(-3a-vp) = ph$.

Problem: Unstable when s near $[e, y]$ (p diverges).

Solution: Do nothing in that case. Define "near" as $|[e, y] - s| < |v * dt|$ (no division needed).

Paper (for a very different audience) at `http://boole.stanford.edu/pub/hpc.pdf`.