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A three-component analytic model of long-term climate change

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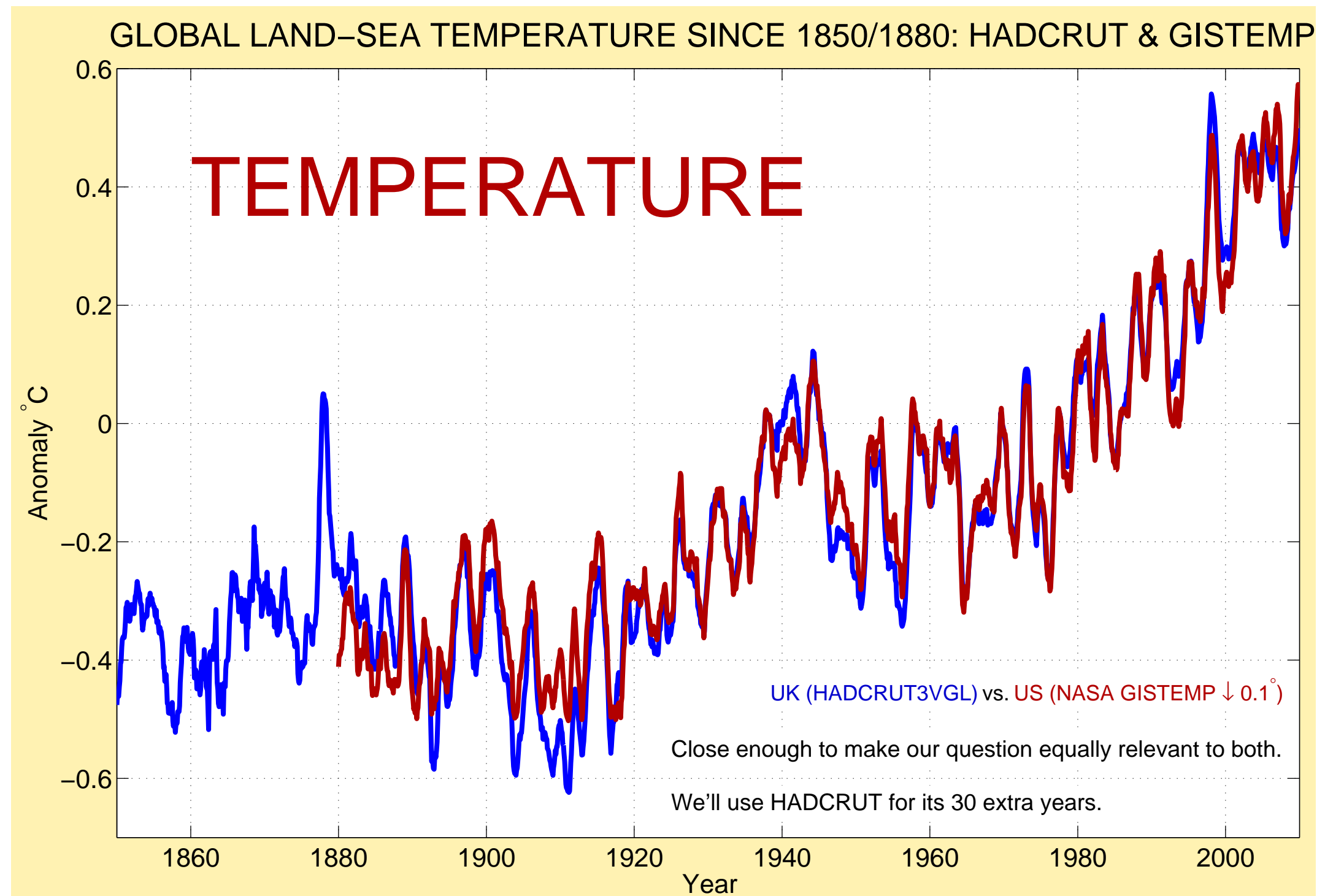
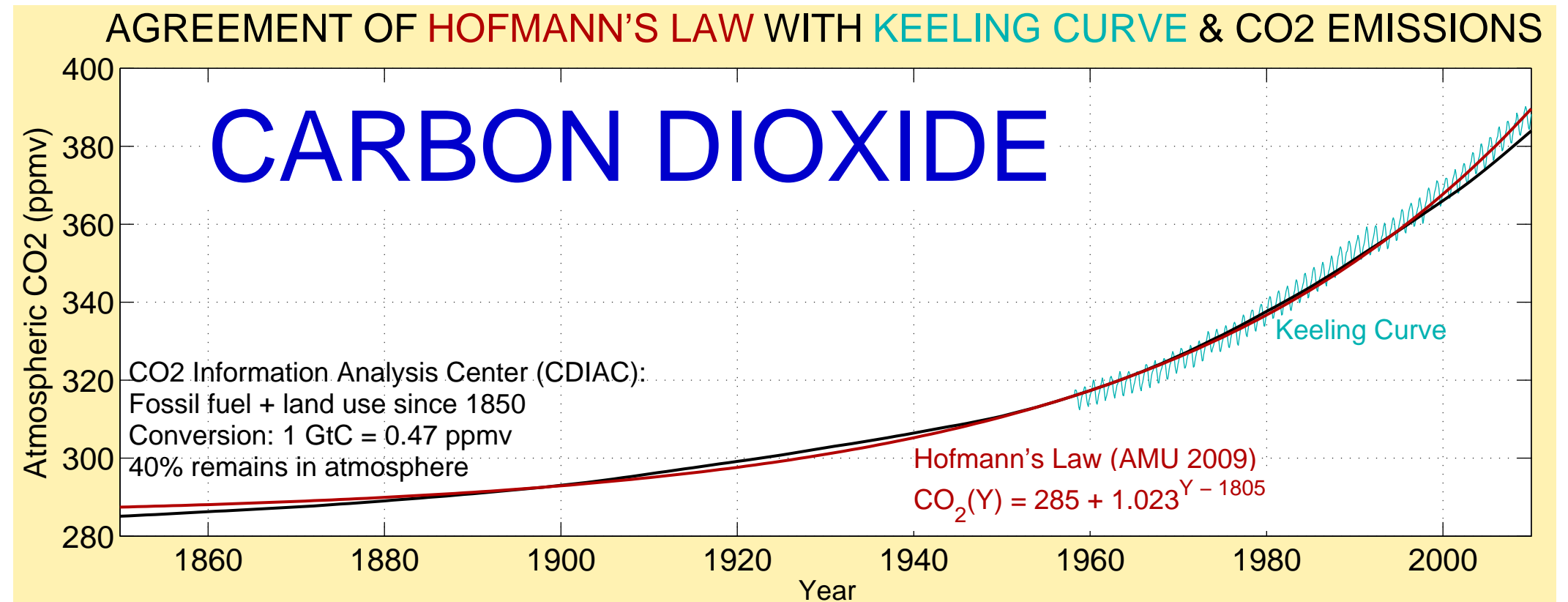
# QUESTION

Why is it that

- $\text{CO}_2$  increases steadily, but
- global temperature fluctuates?

Traditional answers:

- Inadequate emissions controls in fifties and sixties
- Fluctuations in volcanism



# ANALYSIS

We analyze the 1850-2010 Hadley-CRU monthly global land-sea temperature record, HAD for short, as the sum of  $4+2+3+2 = 11$  components, organized into four groups as follows.

|   |   |
|---|---|
| 1. MULTIDECADAL                             | 4. SEASONAL                                       |
| Long-term group:<br>AGW + 2 Osc'ns + UEV    | Very short-term group:<br>Half-yearly + quarterly |
| 2. SOLAR                                    | 3. SUBDECADAL                                     |
| Medium-term group:<br>Hale and Solar cycles | Short-term group:<br>ENSO + Biannual + Annual     |

1. Multidecadal: AGW + PMO + AMO + UEV.

We understand AGW(y), PMO(y), and AMO(y) as analytic functions of the year  $y$ , giving the three respective contributions to the global 1850 – 2010 HADCRUT3VGL temperature record. These are respectively the log of a raised exponential, and two sinusoids; we denote their (analytic) sum by MODEL. We define the *unexplained variance* UEV as the result of passing HAD – MODEL through a triangle filter. We estimate the UEV defined in this way as 0.05% of the total variance of HADCRUT3VGL. That is, the  $r^2$  of this analytic model of long-term climate change is a remarkable 99.95%.

*AGW* We model AGW with what we call the *Arrhenius-Hofmann law* as a function of year  $y$ .

$$AGW(y = 1810 + 20 + 30dp) = 2.83 \log_2(285 + 2^{dp}).$$

(Here  $dp = (y - 1830)/30$  denotes the number of anthropogenic CO<sub>2</sub> doubling periods since 1810+20.) At a 2009 AMU meeting David Hofmann presented his raised-exponential law modeling modern atmospheric CO<sub>2</sub> as a natural base, 285 ppmv in our version, plus an exponentially growing anthropogenic component that was 1 ppmv in the year Indust<sub>0</sub> = 1810 and that has since doubled every 30 years. We add 20 years to 1810 as our least-squares estimate of the effective delay of CO<sub>2</sub>'s thermal impact.

113 years earlier Svante Arrhenius proposed a base-2 logarithmic law for the influence of atmospheric CO<sub>2</sub> on Earth's surface temperature, whose coefficient, here 2.83, we nowadays very loosely call climate sensitivity in units of degrees per doubling of CO<sub>2</sub>.

This is not to say that climate sensitivity has a well-defined value. During the past million years of deglaciations, it took atmospheric CO<sub>2</sub> about half a century to climb 1 part per million by volume (ppmv). At today's rate of climb this is accomplished in a mere two years, and this rate shows no sign of slowing down. There is no reason to suppose that climate sensitivity is the same over this wide range of rates of increase of atmospheric CO<sub>2</sub>.

*PMO, AMO* We model two of the better-known long-period ocean oscillations as sinusoids with respective periods 75 and 50 years and respective amplitudes 0.088 and 0.06 degrees, whose positive-going zero-crossings coincide in 1925. Their sum OSC(y) can be written thus.

$$\text{OSC}(y = 1925 + 24\theta) = 0.088 \sin(2\theta) + 0.06 \sin(3\theta)$$

OSC(y) is then subtracted from the original signal, and the residue is split into seven bands by successively low-pass filtering the low-frequency end of what remains at each stage. Filtering is accomplished by convolution with a triangle filter. The first residue is placed in the long-term group and called the unexplained variance.

## 2. Medium-term: Hale + Solar.

Currently we are in transition from solar cycle 23 to 24. In the even (odd) numbered cycles the magnetic fields of the Sun and Earth are aligned (oppositely aligned). Cycle 23 is oppositely aligned and shows a warming of Earth's surface at both Hale and solar-cycle periodicities.

3,4. Short and very-short: The latter two groups of our four group analysis are simply a continuation of the procedure by which the first two groups were obtained.

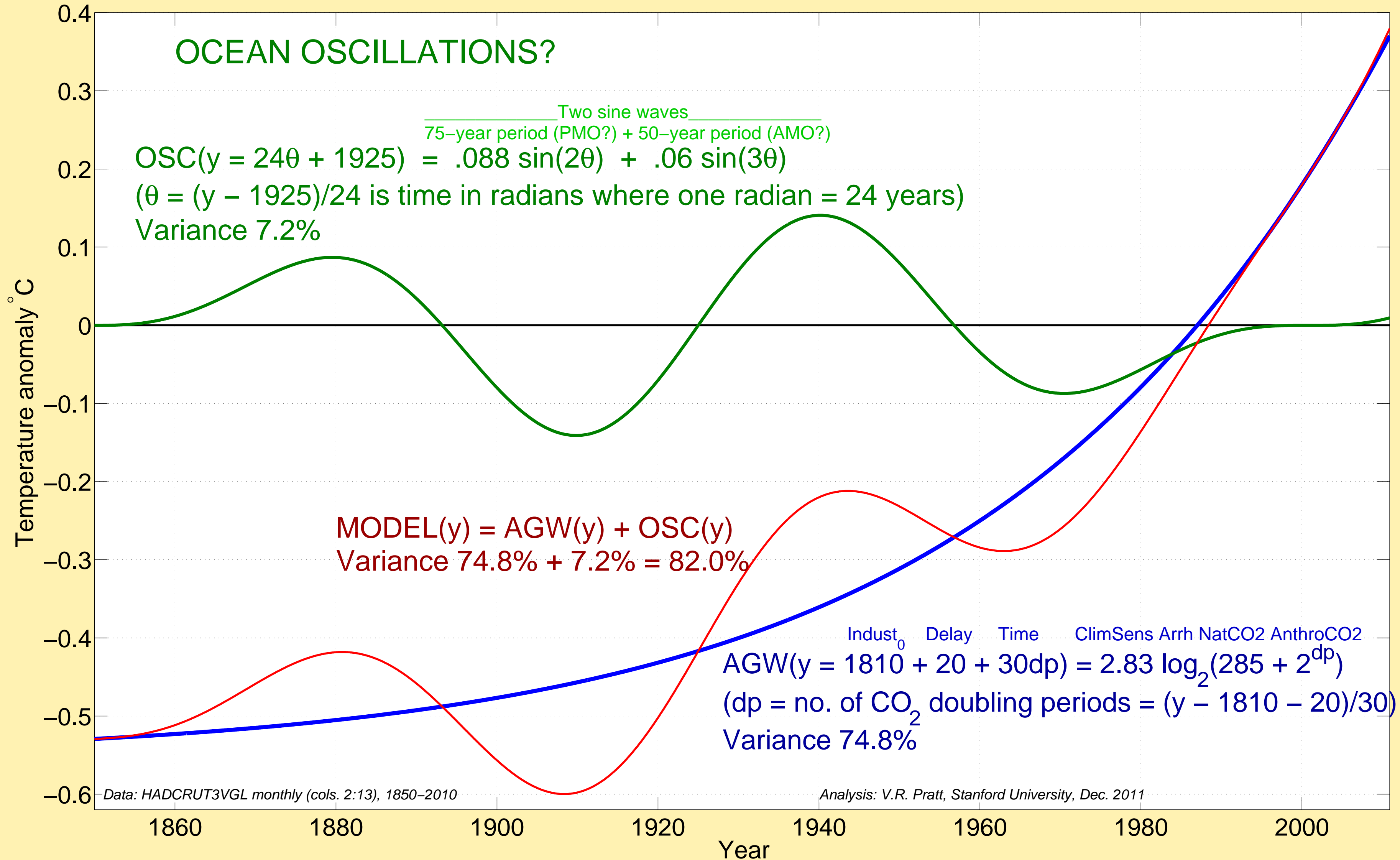
# CONCLUSIONS

1. The Arrhenius-Hofmann law is sufficient to account on its own for any trend in global temperature that could be considered correlated with either human population or advances in technology. Our analysis is unable to distinguish influence of aerosols from influence of greenhouse gases.
2. The AGW signal accounts for 80% of the variance of the whole signal. The three analytically modeled multidecadal signals account for 99.95% of the variance in the bands beyond the 22-year Hale cycle. The dimension of those bands is  $160/22 \approx 7$ , so it is surprising (to us) that our two-dimensional analytic model (the two sine waves) accounts for essentially all of that space.
2. We observe the 11-year solar cycle, but we also observe the 22-year Hale cycle, whose cooling troughs align with the odd-numbered sunspot cycles as shown by the vertical black lines in graph 2.
3. In this analysis the pause in the warming for 2000-2010 is entirely due to the two solar cycles sinking synchronously and strongly, last observed in 1940.
4. The analysis accounts for the sharp peak at 1940 in terms of the alignment of all four of the longest-term signals, namely the two multidecadal sinusoids and the two solar cycles, all of which peak simultaneously at 1940.
5. Since none of the natural signals (all but AGW and UEV) shows any sign of a trend, the responsibility for continued warming during the coming century must rest solely with the AGW signal. AGW(2100) is when the number  $dp$  of anthropogenic CO<sub>2</sub> doubling periods equals  $(2100 - 1830)/30 = 9$ . Hence AGW will contribute  $2.83 \log_2(285 + 2^9) = 27.3$  degrees, which is 2.9 degrees above our present temperature of  $2.83 \log_2(392) = 24.4$  degrees.

# UNCERTAINTIES

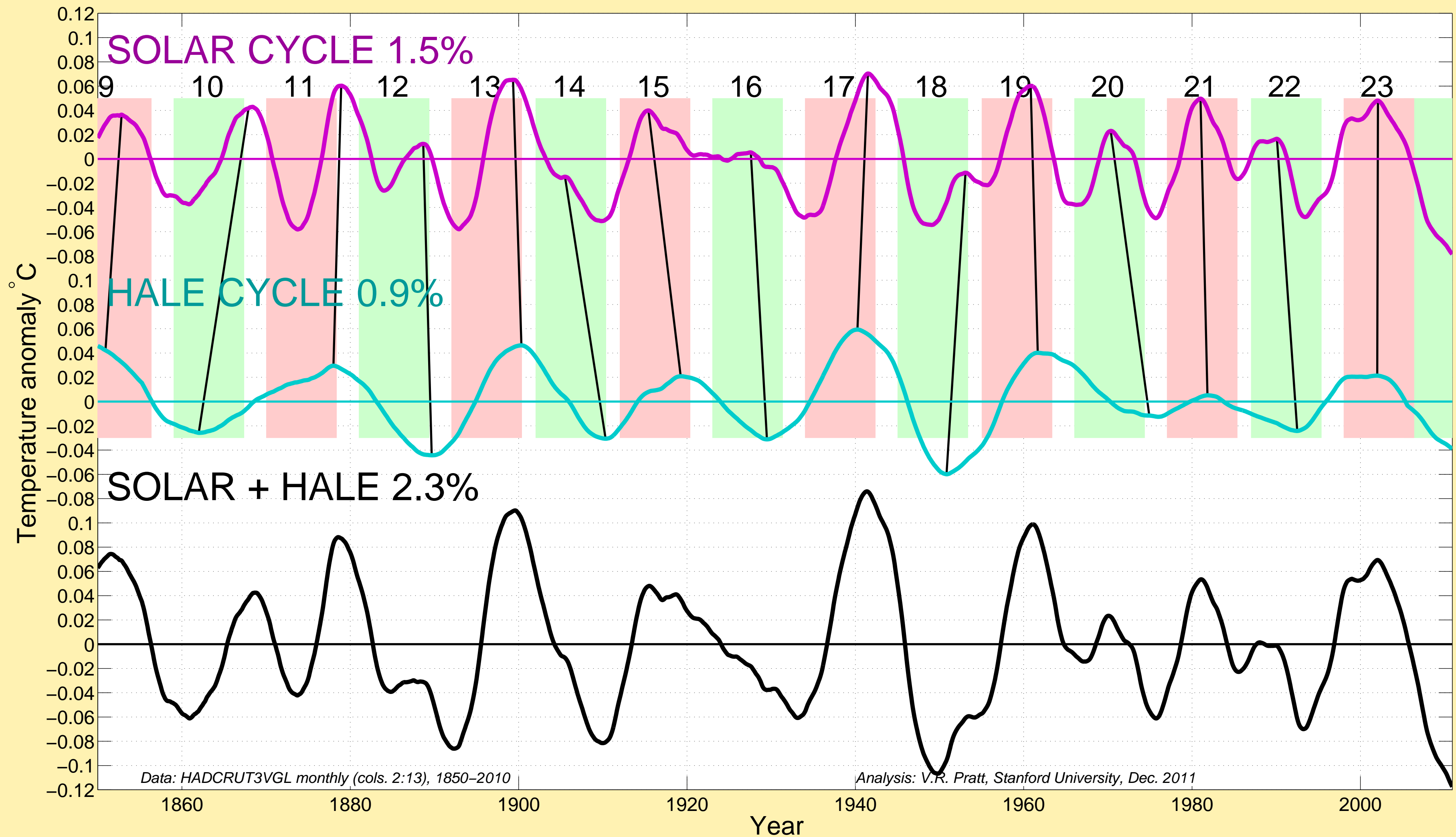
1. This record has not experienced a major tipping point such as release of a substantial portion of tundra methane clathrates, and hence cannot indicate the likely outcome of such an event.
2. Similarly there has been no cessation in CO<sub>2</sub> emissions, another eventuality that this analysis is unsuitable for predicting the consequences of.
3. The climate sensitivity of 2.83 is highly sensitive to both the choice of preindustrial base and of the presumed delay of thermal impact. Raising the former by 5 ppmv or the latter by a decade raises the climate sensitivity by half a degree per doubling. This strong dependence on these two assumed values points up an inherent weakness in empirical estimates of climate sensitivity. It is an excellent question whether other analytic methods are less dependent on such unknowns.
4. Analyses of this kind are in general not unique, and alternative analyses may lead to different conclusions. One approach to judging the relative merits of any such competing conclusions is in terms of the quality of the analysis methodologies supporting those conclusions as they come to hand.

# 1. MULTIDECADAL

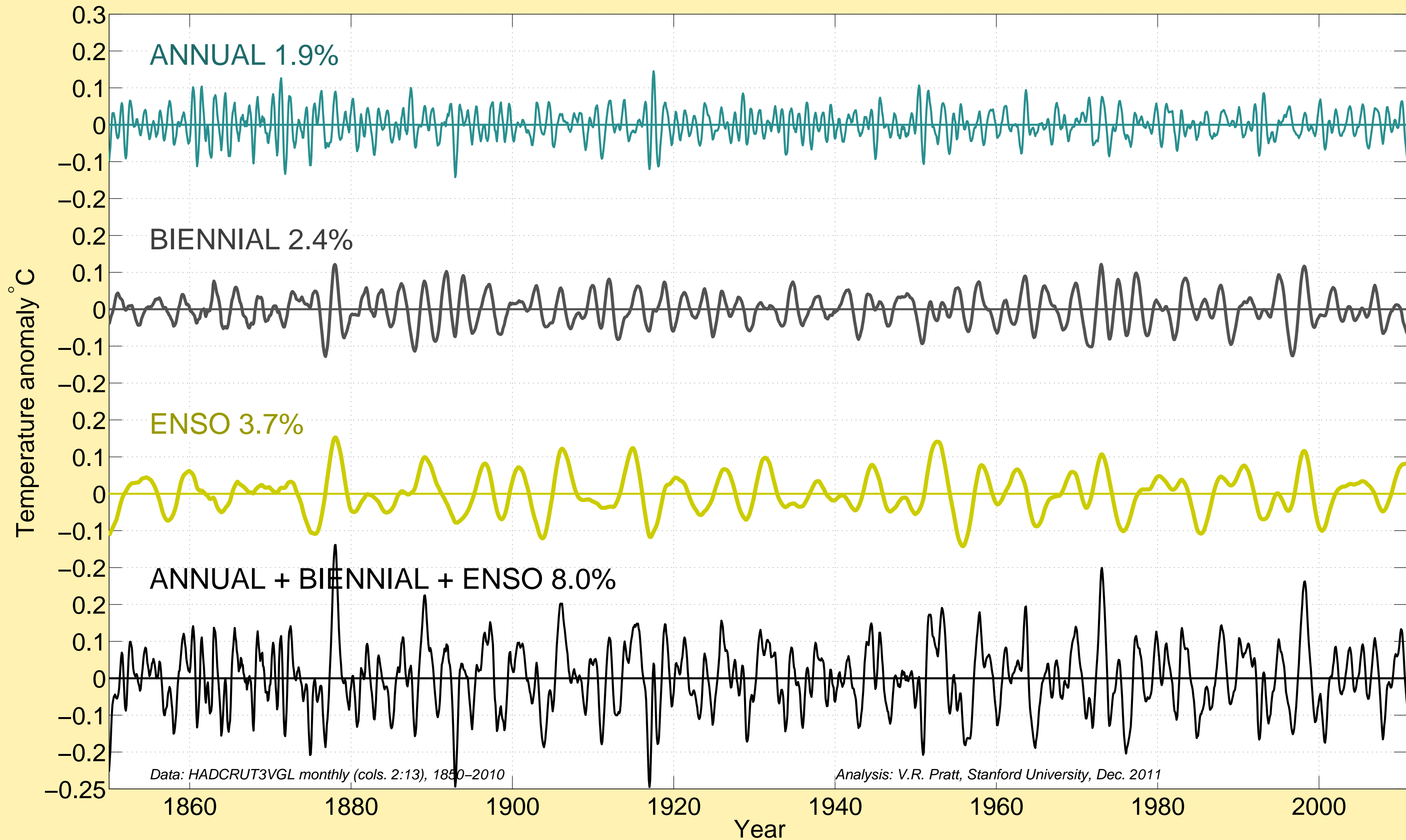




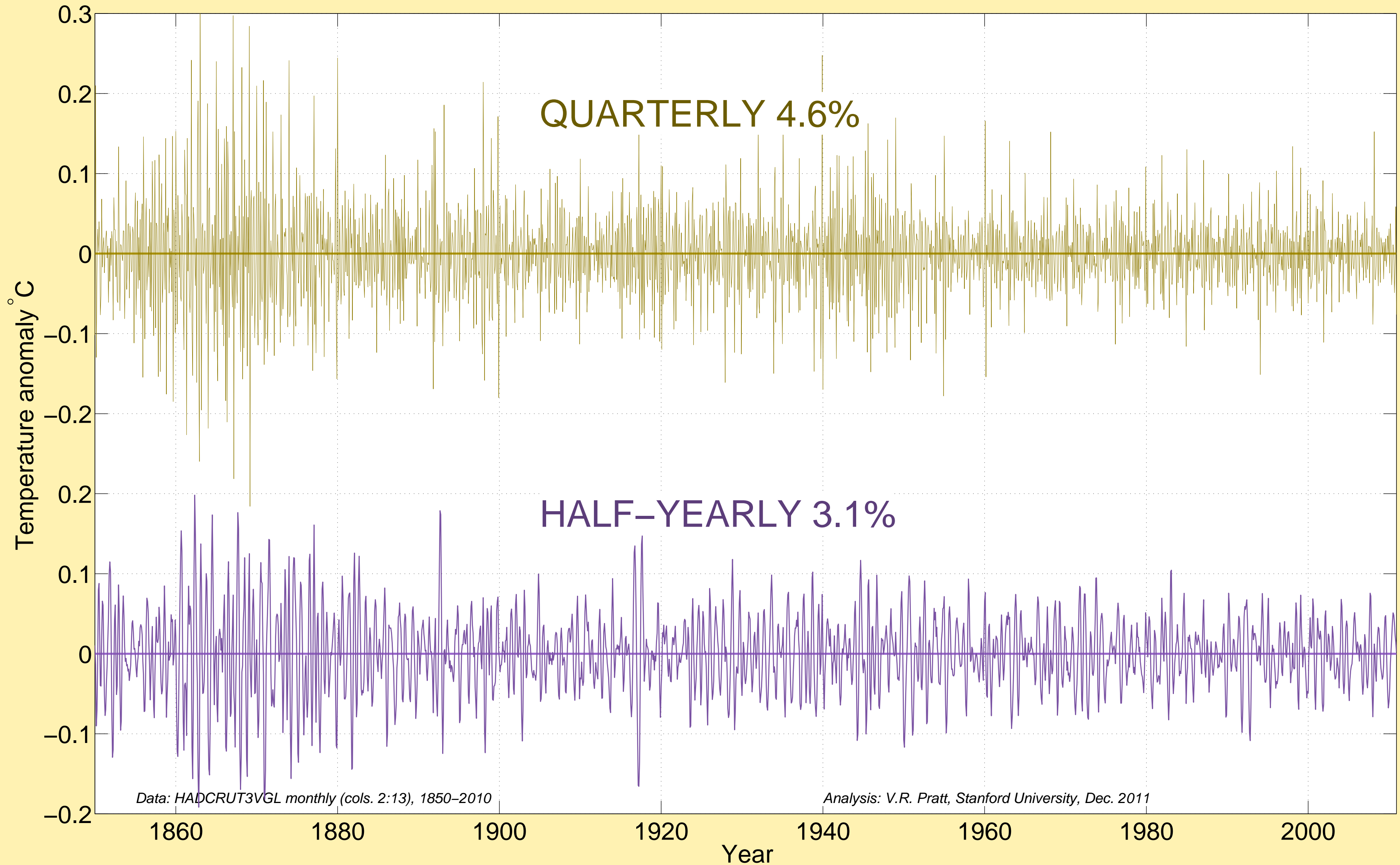
# 2. SOLAR



# 3. SUBDECADAL



# 4. SEASONAL



DISTRIBUTION OF GLOBAL TEMPERATURE VARIANCE  
1850–2010

ANTHROPOGENIC

NATURAL

SHORT  
TERM

LONG  
TERM

AGW 74.8%

QTR 4.6%

HALF 3.1%

ANN 1.9%

2YR 2.4%

ENSO 3.7%

SOL 1.5%

HALE 0.9%

UEV 0.06%

50YR 2.2%

75YR 4.9%

