V. *Note on the Theory of the Greenhouse.* *By* C. G. ABBOT, *Director, Astrophysical Observatory, Smithsonian Institution* *.

IN a paper of the above title † Professor R. W. Wood states that he has compared two "hot-boxes" one having a glass cover, the other a cover of rock salt, but otherwise similar. A glass plate was interposed in the path of the entering sun rays. He observed a maximum temperature of about 55° C. within each box when exposed to the sun. He concludes that the function of the cover is mainly to prevent the loss of heat by convection, rather than the escape of long wave rays, and asks: "Is it therefore necessary to pay much attention to trapped radiation in deducing the temperature of a planet as affected by its atmosphere?"

It may interest some to know that much higher temperatures can be reached within a "hot-box" than that observed by Professor Wood, if precautions are taken to diminish the loss of heat by convection from the warmed outer surface of the cover. On November 4, 1897, the thermometer recorded 118° C. within a circular wooden box 50 centimetres in diameter, 10 centimetres deep, insulated in feathers, covered with three superposed and separated sheets of plate glass and exposed normally to the sun rays in the yard of the Astrophysical Observatory at Washington. The temperature outside was 16° C.

Agreeing with Professor Wood that the main function of the cover of a "hot-box" or "hot-house" is to prevent loss of heat by convection, it is interesting to see if this could be predicted. Published experiments on the cooling of solids in dry air and in vacuum give the relative rates of loss by convection and radiation under known circumstances. Planck's radiation formula for the "black body" enables computations to be made of the losses by radiation for different temperatures of source and sink. The transmission of glass, salt, and the water vapour of the atmosphere, and the effective temperature of the latter are approximately known. I have attempted to compute from such data the relative hindrance which salt and glass covers would interpose to the loss of heat by convection and radiation combined from a "black" surface at 55° C. For the dependence of the temperature of the earth's surface on the atmosphere, some numerical data can be assigned also, and as shown below there is reason to think that "trapping" is more important perhaps than Professor Wood thinks.

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† Phil. Mag. 6th series, vol. xvii. p. 319 (1909).

From an interesting paper of P. Compan * it may be seen that for a blackened copper ball 2 centimetres in diameter cooling from a temperature of 55° C. to nearly "black" surroundings at 0° C., the rate of loss of heat by convection in still dry air at atmospheric pressure is four-thirds as rapid as the simultaneous loss by radiation. In a breeze of 3 metres per second the convection loss becomes 3 times as rapid as in still air, or 4 times as rapid as the loss by radiation. The loss of heat by convection alone is approximately proportional to the difference of temperatures between the source and the sink.

If the covers had been absent in Professor Wood's experiments, the boxes would have been exchanging radiation principally with the water vapour of the lower atmosphere. Experiments of Langley, Rubens and Aschkinass, and others indicate † that less than 10 per cent. of the radiation from the earth's surface can penetrate the water vapour of the atmosphere above a coast station like Baltimore. Hence the water vapour of the atmosphere can be considered as practically a "black body" for rays of great wave-length. The effective temperature of the water-vapour layers with which a coverless "hot-box" would have been exchanging radiation may be estimated at 0° C. If glass is interposed the radiation is entirely cut off. If a rock-salt plate 1 centimetre thick is interposed between the body at 55° C. and surroundings at 0° C., the absorption of the plate is about 19 per cent.‡ and the reflexion probably nearly 10 per cent. more, so that the transmission may be reckoned at about 70 per cent.

In combining the preceding results with those of Compan we will at first neglect the loss of heat by convection from the outside of the cover. We may assume the temperature of the air just outside the "hot-box" to be 15° C., and also that Newton's law of cooling is applicable to the convection loss. In still air with no cover the rate of loss of heat towards the front by convection and radiation combined is proportional to:

$$\frac{40}{55} \times 133 + 100 = 197.$$

With glass cover: $0 + 100 \times 0{\cdot}00 = 0.$

With salt cover: $0 + 100 \times 0{\cdot}70 = 70.$

* *Annales de Chimie et de Physique*, t. xxvi. pp. 488–574 (1902).

† See Annals, Astrophysical Observatory, Smithsonian Institution, vol. ii. pp. 167–172.

‡ See Kayser's *Handbuch d. Spectroscopie*, vol. iv. p. 485, and Planck's formula of radiation.

Thus of the heat which would have escaped toward the front from a coverless box at 55° C. in still moist air at 15° C., the salt hinders $\frac{197-70}{197}=65$ per cent. as much as the glass. Remembering, however, that owing to its higher absorbing power for long wave rays the glass will be warmed more than the salt, the convection loss from the outside of the warmed cover will be greater for glass than for salt, so that the efficiency of a salt cover may be much more than 65 per cent. of that of one of glass. The convection loss from the front of the cover is a considerable factor, for in the "hot-box" tried at this observatory the front of the inner glass cover became too hot to handle and often cracked with the heat.

In view of these figures we may agree with Professor Wood that a salt cover * is nearly as efficient as a glass one for a "hot-box," although it would seem strange that he observed no difference at all. Perhaps in spite of the glass filter the cover-glass obstructed the entering sun rays more than salt. But is not the case quite different with a planet?

Let us take the mean temperature of the earth's surface at 14° C., the mean effective temperature of the water-vapour layers to which it principally radiates as 0° C., the temperature of space as −273° C. Then the rates of escape of heat from the surface by radiation, first with the water-vapour layer interposed, and, second, imagining the air to be completely transparent to earth rays, would be in the ratio of 19 to 100 according to Planck's formula. It is very difficult to estimate how fast the heat of the earth's surface escapes by convection, because neither the difference of temperature between the surface and the air nor the rate of motion of the air is well known. But if for the sake of discussion we suppose a temperature difference of 10° C. and a velocity of 3 metres per second, the rate of convection loss comes out only 0·54 as great as the rate at which heat would escape by radiation if the air was no hindrance. This assumed convection loss is 2·8 times as great, on the other hand, as the estimated rate of escape of heat by radiation to the water-vapour layers at 0° C. In other words, according to this estimate the convection is the main agent in removing heat from the earth's surface as things are, but would be only a small factor if the air was transparent to long-wave rays.

If these figures represent at all the order of magnitudes of the quantities there can be no doubt, I think, that the atmosphere is important as a trapping agent to increase the earth's surface temperature.

* A salt cover, however, is better than a perfectly transparent one.

A fair estimate of the actual increase of the earth's temperature due to "trapping" has been made. Imagine a perfectly "black" and rapidly rotating planet of the earth's dimensions, situated beyond the orbit of the earth at such a distance from the sun that the radiation absorbed by it would be equal to that available to be absorbed by the earth, allowing for the reflexion of clouds, et cetera. Such a planet would assume the temperature of −17° C., whereas the real earth has a mean temperature of +14° C.* The difference, 31° C., is attributable to three causes:—(1) The imperfect "blackness" of the earth. (2) The "blanket effect" of the atmosphere. (3) The warming of the earth's surface by radio-active substances and internal heat. Remembering that the earth is mainly water covered, it must be almost "perfectly black" for long-wave rays. I myself regard the conduction of internal heat and that of radio-active substances as negligible. This would leave the full 31° as due to the "blanket effect."

If there were no water on the earth, the emissive power of its surface for long-wave rays would be less. Also, on account of the absence of clouds, its absorption of solar rays would be greater. The two differences would perhaps more than counterbalance the loss of the "blanketing effect," so that the mean temperature of the earth without water would perhaps be rather higher than now, but much less uniform, ranging from above present temperatures by day to far below 0° C. by night.

VI. *On the Steady Flow of an Incompressible Viscous Fluid through a Circular Tube with Uniformly Converging Boundaries.* By A. H. Gibson, *M.Sc., Lecturer in Hydraulics in the Manchester University* †.

THE general equations of motion for the three-dimensional flow of an incompressible viscous fluid are ‡:—

$$\left.\begin{aligned}
-\frac{dp}{dx} &= -\mu\left\{\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right\}+\frac{\varpi}{g}\left\{u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}+\frac{\partial u}{\partial t}\right\}\\
-\frac{dp}{dy} &= -\mu\left\{\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right\}+\frac{\varpi}{g}\left\{u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}+\frac{\partial v}{\partial t}\right\}\\
-\frac{dp}{dz} &= -\mu\left\{\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right\}+\frac{\varpi}{g}\left\{u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}+\frac{\partial w}{\partial t}\right\}
\end{aligned}\right\} \quad (1)$$

* Annals, Astrophysical Observatory, vol. ii. p. 175.

† Communicated by the Author.

‡ 'Hydraulics,' Gibson, p. 65.